Babbitt's *Canonical Form* Revisited: Codes and Metaphors for Epistemology

Joshua Banks Mailman

Perspectives of New Music, Volume 58, Number 2, Summer 2020, pp. 215-245 (Article)

Published by Perspectives of New Music

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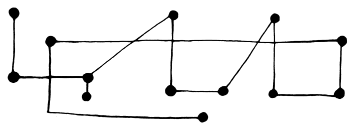

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Even among Milton Babbitt's composition titles (such as *It takes Twelve to Tango*, *Whirled Series*, and the *Joy of More Sextets*) his title *Canonical Form* is unusually mysterious, not for being abstruse or impenetrable, but rather for being so multiply penetrable; that is, for being so easily unwrapped to reveal several possible meanings. Until recently, the titles of Babbitt's compositions have not been subject to much scrutiny, usually being treated merely as casual diversions, prompting mere casual speculation. Alison Maggart's (2017; 2021) recent work, however, delves into aspects of Babbitt's biography, identity, and personality to develop an unprecedented hermeneutical account of his music, foregrounding Babbitt's predilection for wordplay in the form of puns, considering the possible power dynamics of humor, Babbitt's Jewish identity, and his nostalgia for baseball. The present essay pursues such a hermeneutical account, focusing on Babbitt's solo piano work *Canonical Form* (1983).

The third of Joseph Dubiel's "Three Essays on Milton Babbitt" (1990; 1991; 1992) addresses the overall grandeur of Babbitt's *Canonical Form*, suggesting some possible meanings of the title, which provide part of the foundation that my remarks build on. Yet since the time of Dubiel's writing, not only have Babbitt's titles begun to be scrutinized more closely (Maggart 2017; 2021), but also a number of analytical technologies and attitudes have arisen which enable, or underwrite, an even broader spectrum of interpretations than was perhaps possible, or tenable. For instance, Gilles Fauconnier and Mark Turner's (1998; 2002) *conceptual blending* theory, including the analytical technology of *conceptual integration networks* (CINs), is enlisted to model the multifaceted, multivalent nature of Babbitt's music and titles.[1] It provides a framework for interpreting various analytical results, as I've demonstrated before. For instance, as I showed previously (Mailman 2020) and show below, Schenkerian analysis can be deployed to reveal tonality in Babbitt's music, such that it can be read as a conceptual blend of serial and tonal syntaxes. Later in the analyses below I also apply, to pitch registers, both oscillator modeling and David Lewin's (1995) technique of binary state operators, which he had applied specifically to rhythms and instrumentation in two other Babbitt works. Finally I present a technique of cumulative pattern matching (across domains of pitch class and pitch register) based on Boolean functions (Boolean algebra) which reveals a hidden meaning in Babbitt's title. Yet another arises from an important matrix pattern in linear algebra.

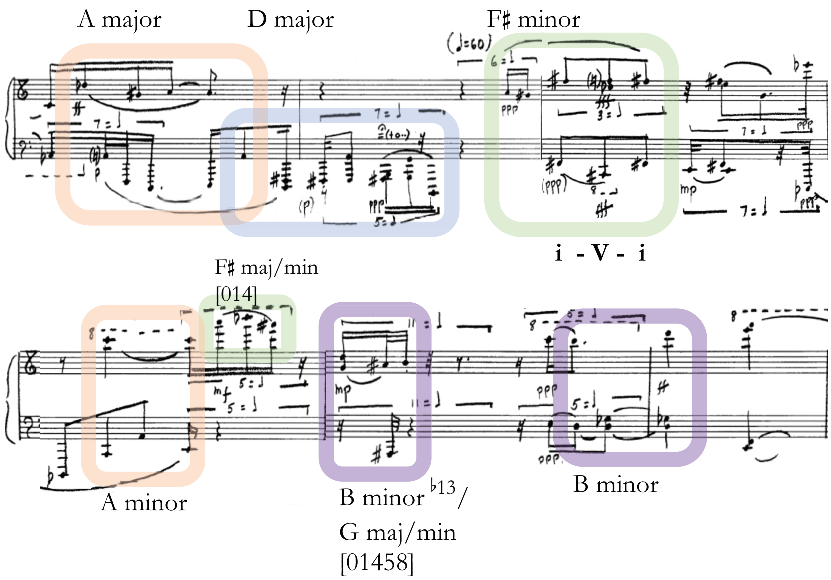
Ultimately, I suggest, as one aspect of Babbitt's compositional poetics, a certain pragmatism regarding ontologies in music theoretical discourse. That is, this piano work and its title, taken together, endorse a certain pragmatism as instrumental to the progress of musical inquiry.

Canonicity

One of the possible meanings of the title "canonical form" that is at least worth considering is the work's engagement with canonical repertoire, not in terms of anything as overt as quoting, but rather through subtler means. In this work, and others from the 1980s, Babbitt's pitch configurations dance with the tonal languages of canonic repertoire, by eluding to brief triadic chord progressions and even longer voice-leading motions.

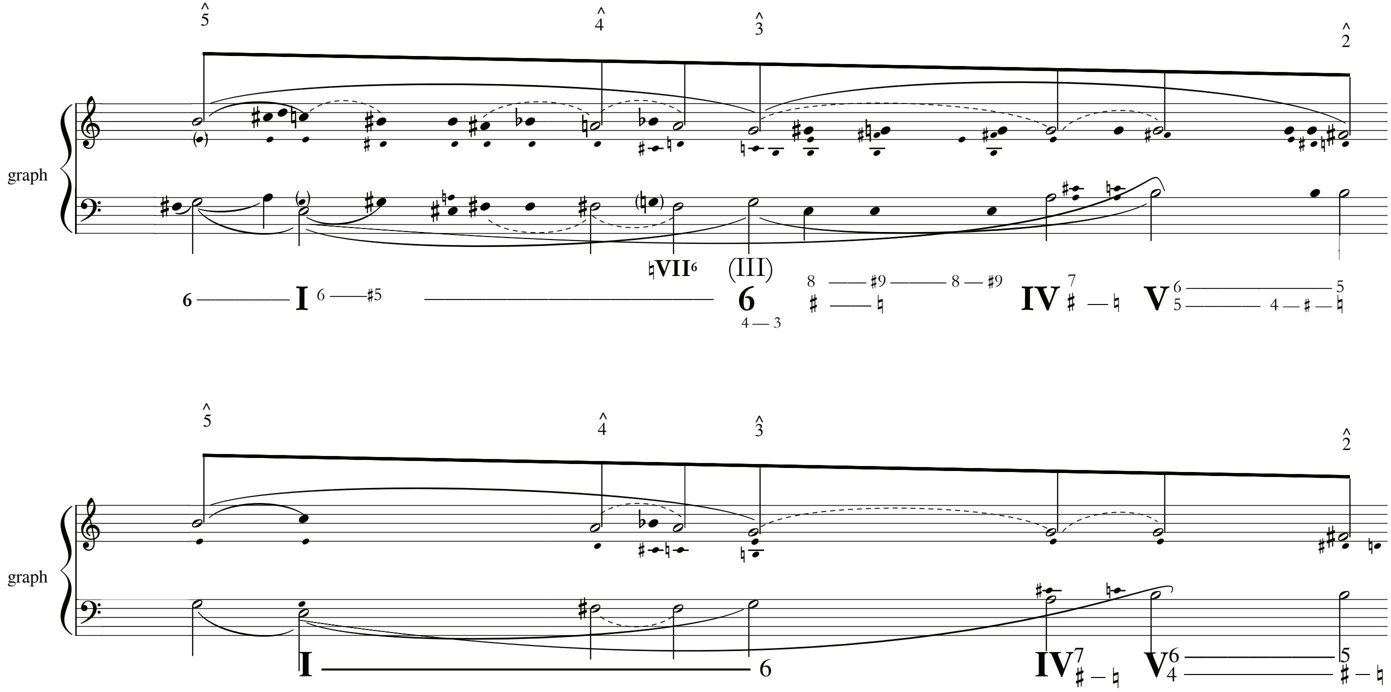
Canonical Form is based on the same array as *Whirled Series*, Quartet No. 5, and *Around the Horn* (Mead 1994; Dubiel 1997), an array whose row features three consonant triads as consecutive segments. Furthermore these works are based on superarrays, which enable a surprisingly vast flexibility in aligning pitches to create various configurations (Mailman 2019). This is because these superarrays feature three or more simultaneous arrays whose ordering relative to each other is undetermined. Thus Babbitt can choose to align pitches from different array lynes, not only to forge consonant triads but also to suggest cadential tonal gestures and, in some cases, longer chromatic tonal progressions.

A passage near the beginning of *Canonical Form* exemplifies this strikingly. Shown in Example 1, a perfect authentic (I-V-I) cadence in F♯ minor launches m. 21, prepared by a melodic lead-in (A and G♯ from m. 20). In m. 22, six pitches, drawn from two arrays, assemble to arpeggiate an A major-minor tetrachord. The next two measures use eight pitches from all three arrays to suggest B minor, consorting with three diatonically related pitches (G, C, and E). Just prior to this, mm. 19–20 configure six pitches to suggest A major, and then a D major triad in the lowest register. Note that the chords suggested (A major, D major, and B minor) are all diatonic to F♯ minor (though many other non-diatonic pitches occur during these passages). So they follow the rule (*canon*) for inclusion in that tonal scale and, as consonant triads, are archetypal (*canonic*) tonal sonorities. The allusions to tonality relate to the work's title. Moreover, as I recently showed (Mailman 2020),



EXAMPLE 1: BABBITT'S *CANONICAL FORM* (1983), MM. 19–24, CADENTIAL GESTURE IN MM. 20–21; REFER TO EXAMPLE 1 AUDIO (A) AND (B) AT WWW.PERSPECTIVESOFNEWMUSIC.ORG/SOUNDEXX

EXAMPLE 2A AND B: *CANONICAL FORM* (1983), MM. 200–03, AND TONAL VOICE-LEADING GRAPH OF THE EXCERPT; REFER TO EXAMPLE 2 AUDIO (A), (B.1, B.2), AND EXAMPLE 2 VIDEOS (A-B.1) AND (A-B.2), AT WWW.PERSPECTIVESOFNEWMUSIC.ORG/SOUNDEXX



Example 2b and c: shallower and deeper tonal voice-leading graphs of *Canonical Form* (1983), mm. 200–03;
refer to example 2 audio (b.1, b.2), and example 2 video (a-b-c),
at www.perspectivesofnewmusic.org/soundexx

a number of passages in *Canonical Form* (as well as *Whirled Series* and others) can be read as tonal phrases, manifesting an *Ursatz* progression. Example 2 shows such a reading for mm. 200–03, with an interrupted *Urlinie* descent in E minor.[2] In forging such a tonal reading of any such passage, we have to observe some flexibility in regard to register, because, after all, these surfaces are forged from twelve-tone serial arrays, which Babbitt deploys according to a global registral distribution decided in advance of composing each passage. Nevertheless, the subtle tonality shines through. In hearing such passages as tonal, the variegated rhythms, voice-leading displacements, and chromatic inflections, come off as something akin to jazz "comping."

Babbitt engaging notions of canonicity by weaving subtle tonal inflections into his serially-based surfaces is certainly witty; since the surface is based on a twelve-tone serial superarray *and* can be heard as tonal, it is a sonic synthesis or fusion of two meanings: a musical pun. I have elsewhere (Mailman 2020) called this *portmantonality* (*portmanteaunality*). Yet, such *portmantonality* seems prevalent throughout his 1980s compositions, such as *Whirled Series*, *Around the Horn*, and Quartet No. 5. So this reference to canonicity isn't unique to this specific piano work.

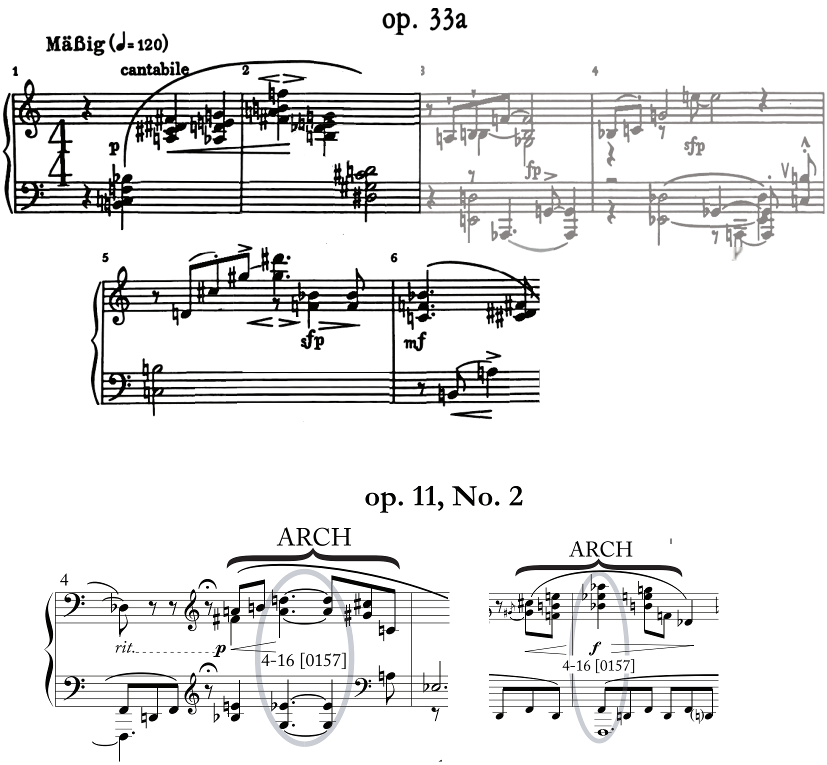
Moreover, this notion of canonicity doesn't engage the full two-word title *Canonical Form*, as there is nothing specifically form-related about tonality, tonal inflections, triadic harmony, tonicity, or Schenkerian voice-leading paradigms. Thus, as interesting and important as it is, this connection to the title *Canonical Form* doesn't quite rise to the level of wordplay that Babbitt persistently infuses into his compositional titles.

Canonical Form as Wordplay

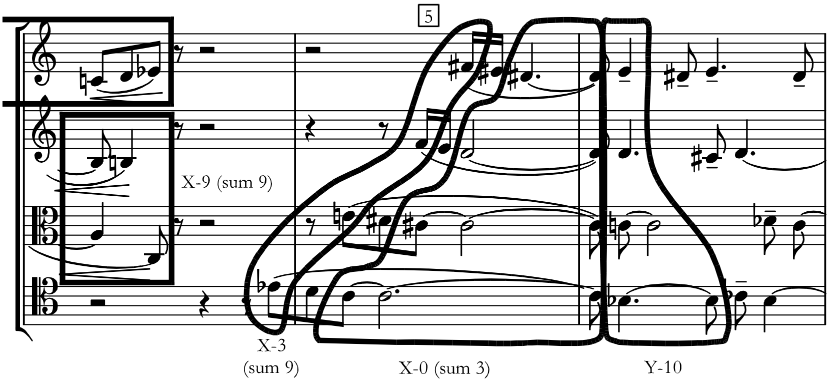
Dubiel's (1992) interpretation of the title is basically that the formal shape of the work is determined by adherence to a list of registral combinations (thus conforming to a *rule*, or *canon*), which is presented in palindrome and thus relates to canonical practices of formal shaping in canonical classical musical works, namely the "arch" form. Not only can these ideas withstand some development, which I'll undertake, but there's another meaning of *canonical form* which seems to have been overlooked, a meaning that is more arcane yet much more philosophically suggestive with regard to the pragmatics of music as a field in general.

Active registers	Sections																							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
High			H	H	H		H		H	H	H	H	H	H	H	H		H		H	H	H		
Middle		M	M	M	M	M		M		M	M			M	M		M		M	M	M	M	M	
Low	L	L	L		L	L	L				L	L	L	L				L	L	L		L	L	L

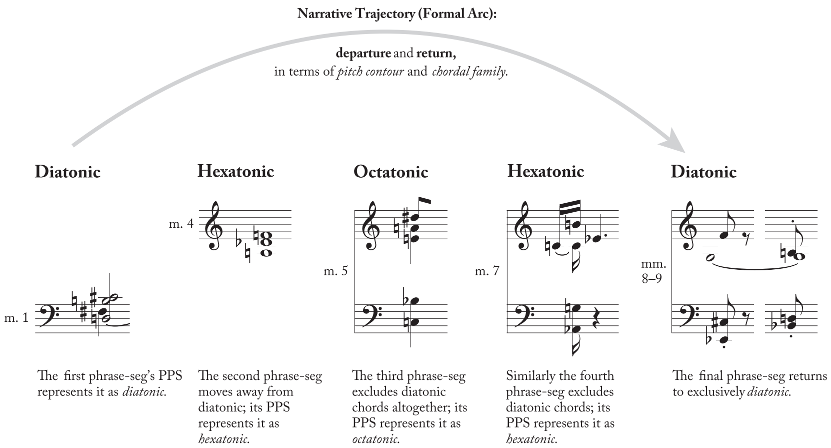
EXAMPLE 3: *CANONICAL FORM* SERIES OF REGISTERS (LOW, MIDDLE, HIGH), FROM DUBIEL (1992)



EXAMPLE 4: LOCAL ARCH FORM GESTURES IN SCHOENBERG'S OP. 33A (RENOTATED) AND OP. 11, NO. 2 (ANNOTATIONS FROM MAILMAN [2015])



EXAMPLE: 5: LOCAL ARCH FORM IN BARTOK'S QUARTET NO. 4, MOVT. 1 (ANNOTATIONS FROM ANTOKOLETZ [1984])

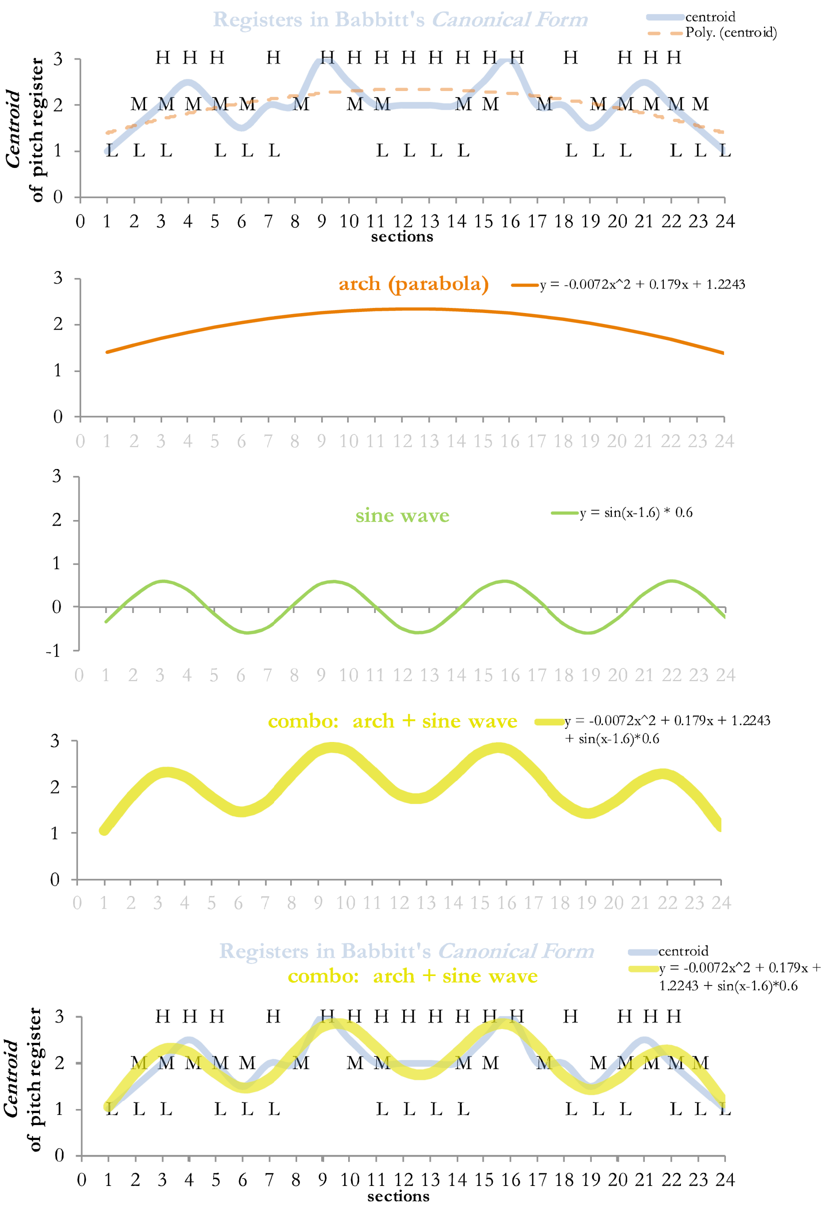


EXAMPLE 17. *Chronological narrative of* Peak-Point Sonorities *(PPSs)*.

EXAMPLE 6: LONG-RANGE ARCH FORM THROUGH A COORDINATION OF REGISTRAL TRAJECTORY AND COLLECTIONAL TYPE IN SCHOENBERG'S OP. 19, NO. 3 (MAILMAN 2015)

Although Dubiel doesn't directly draw the connection, perhaps it's understood that the arch-form interpretation is helped by the fact that the work starts by incrementally ascending in register and ends by descending, as shown in Example 3. He does mention that the lowest register is heard by many as "somehow generative of the entire piece." Indeed, the idea of being generated from "the depths," primordially as it were, *is* canonical enough to be the entire basis of Wagner's epic tetralogy-launching *Rheingold* prelude, the ultimate from-the-depths *ex nihilo* design (Darcy 1993). And the canonicity of these from-the-depths arch-forms extends into the modernist era, over the short range in Schoenberg's Op. 33a and Op. 11, no. 2, and Bartok's Quartet No. 4, shown in Examples 3, 4, and 5; also in the longer range in the outer movements (prelude and march) of Berg's Op. 6 Orchestral Pieces, and even (through a coordination of registral trajectory and collectional type) in Schoenberg's Op. 19, no. 3 (Mailman 2015), reproduced in Example 6. Indeed in the early twentieth century Henri Bergson's *élan vital*, from primordial ooze, to life, and back to dust ("from ashes to ashes, dust to dust"), underwrites Boris Asafyev's (1930) Soviet era theory of musical form, and Gustav Freytag's (1900) *pyramid of dramatic structure* (although Babbitt's differs by featuring a central plateau). It is deeply canonical, embedded as it were in musical culture, dramatic design, and beyond.

Yet it is not the only deeply canonical form for music. We might consider an oscillating wave equally canonical, as it is the vibrational basis of pitch itself as well as the flow of heard meter.[3] Indeed the trajectory of registers in Babbitt's *Canonical Form* is not at all a simple arch-shape. A plausible way to track the rising and falling flow of registers is in terms of the *centroid* or average registral height, so that for instance the higher register alone (H) counts as higher than the middle register together with the high register (MH) and that counts as higher than all three together (LMH). As seen in Example 7A, the trajectory of registral centroids is an oscillating wave; more precisely it's an oscillating wave overlaid on an arch. The arch shape is modeled as an inverted parabola and the wave oscillation as a sine function, shown in Examples 7B and 7C. As in Example 7D and 7E, the inverted parabola and sine function added together closely model the registral trajectory of *Canonical Form*, as if their addition constitutes a *rule* for creating the form. The dynamic form (based on a trajectory of registers) is more than doubly canonical, a synthesis of two formal shapes that are canonical in music, combined by a rule (*canon*) of arithmetic addition.



EXAMPLE 7: MODELING THE TRAJECTORY OF REGISTERS IN *CANONICAL FORM*
(A) REGISTRAL CENTROID; (B) ARCH (AS PARABOLA); (C) SINE WAVE;
(D) COMBINATION: ARCH + SINE WAVE;
(E) COMPARISON OF CENTROID TO COMBO

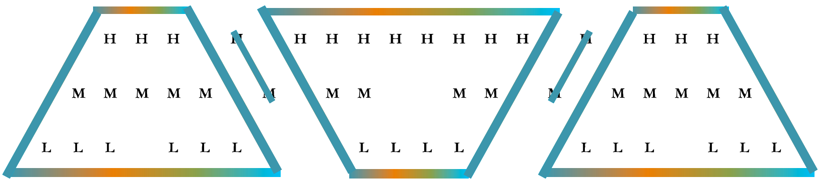
sections	Script of generative rules	Active operators	Binary state operators	Registral state represented with literals and complements
0				$\bar{L}\ \bar{M}\ \bar{H}$
	1. Apply the three smooth operators in ascending order: ⟨100⟩, ⟨010⟩, ⟨001⟩.		≀ ⟨1 0 0⟩	
1		L		L $\bar{M}\ \bar{H}$
			≀ ⟨0 1 0⟩	
2		L M		L M $\bar{H}$
			≀ ⟨0 0 1⟩	
3		L M H		L M H
			≀ ⟨1 0 0⟩	
4	local centroid climax	M H		$\bar{L}$ M H
			≀ ⟨1 0 0⟩	
5	2. Present the states in reverse order.	L M H		L M H
			≀ ⟨0 0 1⟩	
6		L M		L M $\bar{H}$
			≀ ⟨0 1 1⟩	
7	3. Use stronger operators ⟨011⟩ and ⟨111⟩ to create any states missing so far (complete the aggregate of states).	L H		L $\bar{M}$ H
			≀ ⟨1 1 1⟩	
8		M		$\bar{L}$ M $\bar{H}$
			≀ ⟨0 1 1⟩	
9		H		$\bar{L}\ \bar{M}$ H
			≀ ⟨0 1 0⟩	
10	4. Apply smooth operators ⟨100⟩, ⟨010⟩, which maintain the highest register, H, while lowering the centroid.	M H		$\bar{L}$ M H
			≀ ⟨1 0 0⟩	
11		L M H		L M H
			≀ ⟨0 1 0⟩	
12		L H		L $\bar{M}$ H
			≀ ⟨0 0 0⟩	
13	5. Reverse the entire 12-section pattern, to create a long-range palindrome lasting for 24 sections.	L H		L $\bar{M}$ H
			≀ ⟨0 1 0⟩	
14		L M H		L M H
			≀ ⟨1 0 0⟩	
15		M H		$\bar{L}$ M H
			≀ ⟨0 1 0⟩	
16		H		$\bar{L}\ \bar{M}$ H
			≀ ⟨0 1 1⟩	
17		M		$\bar{L}$ M $\bar{H}$
			≀ ⟨1 1 1⟩	
18		L H		L $\bar{M}$ H
			≀ ⟨0 1 1⟩	
19		L M		L M $\bar{H}$
			≀ ⟨0 0 1⟩	
20		L M H		L M H
			≀ ⟨1 0 0⟩	
21		M H		$\bar{L}$ M H
			≀ ⟨1 0 0⟩	
22		L M H		L M H
			≀ ⟨0 0 1⟩	
23		L M		L M $\bar{H}$
			≀ ⟨0 1 0⟩	
24		L		L $\bar{M}\ \bar{H}$
			⟨1 0 0⟩	
				$\bar{L}\ \bar{M}\ \bar{H}$

EXAMPLE 8: A SCRIPT OF FIVE RULES FOR GENERATING THE REGISTRAL PATTERN OF *CANONICAL FORM*, USING BINARY STATE OPERATORS

Moreover the precise sequence of *Canonical Form*'s active registers can be generated by a rule—or, more specifically, a script—involving five rules applied in order. To see how this works we view each register as a binary state (active vs. inactive) and apply three-bit binary operators to each state, where 1 reverses the state of the register and 0 leaves it unchanged. (It's a technique developed by David Lewin [1995] and applied by me to Schoenberg [Mailman 2015]). Let an overline signify that a register is inactive and define the time before the piece begins as a section with no registers active, thus $\overline{L}.\overline{M}.\overline{H}$. The script of rules is shown in Example 8: (1) Apply the three smoothest operators in ascending order: ⟨100⟩, ⟨010⟩, ⟨001⟩ (they are smoothest because they only affect one register at a time) to reach the first centroid climax $\overline{L}.M.H$; (2) Using the local centroid climax (at section 4) as an axis of reflection, present the states in reverse order (retrograde), creating a palindrome; (3) Use stronger operators ⟨011⟩ and ⟨111⟩ to create any states missing so far (complete the aggregate of states); (4) apply smoother operators that maintain the highest register while lowering the centroid; (5) reverse the entire pattern so far, to create a long-range palindrome. These moves are a recipe to parcel out, through binary switching, slow waves of increasing climax and recession.

What's hidden in plain sight of course is that almost all twelve-tone compositions are—at least on some abstract level—instances of that archetypal form of imitative polyphony: the *canon*. For instance, as Dubiel (1992) alerts us, the lowest register array is the retrograde of the two higher arrays. This makes the entire piece a crab canon, so named, presumably, because of a crab's typical side-to-side movement in either lateral direction. Out of sheer irony or serendipity, or as some mysterious circuitous link between temporal symmetry and crustacean ambidextrous morphology, the orthochronic picture of Babbitt's registral pattern actually resembles the modular oblong bilateral symmetry of a crab viewed head-on, as shown in Example 9. What emerges is the suggestion that musical form can arise out of the contrapuntal technique of canon, or more generally from a rule (or script of rules) as Babbitt's expanded version of serialism illustrates, furtively though conventionally in terms of strings of pitch classes and, more starkly but unorthodoxically, in terms of patterns of active vs. nonactive pitch registers. The flexibility of canons (generative rules) in music is thus illustrated uncanonically (using means beyond any pre-established convention).

Yet what I'm most interested in is another meaning in Babbitt's title that engages not just both words *canonical* and *form* but rather the entire two-word combination *canonical form* as well as Babbitt's involvement with discrete mathematics. And this meaning has even broader and deeper resonances poetically, didactically, perhaps even philosophically.



EXAMPLE 9: THE PALINDROME OF REGISTERS IN BABBITT'S *CANONICAL FORM*, RENDERED VISUALLY, SOMEWHAT RESEMBLES THE FRONT VIEW OF THE SEGMENTED BILATERAL SYMMETRY OF A CRAB, WHOSE SIDE-TO-SIDE LATERAL MOVEMENT IS PRESUMABLY WHAT A CRAB CANON IS NAMED AFTER

Some technical setup is required, as this meaning of *canonical form* is based in mathematical logic, although its import is broader.

In mathematical logic, a Boolean function is a logical expression involving a set of true or false (on or off) variables usually represented by letters, for instance P ∨ Q means "either P or Q (or both) is true"; whereas, P ⇒ Q means "if P is true, then Q is true" (P only if Q). Each

$\overline{H}.(M \Rightarrow L)$

L	M	H	Output	Minterm
0	0	0	1	$\overline{L}.\overline{M}.\overline{H}$
1	0	0	1	$L.\overline{M}.\overline{H}$
0	1	0	0	
0	0	1	0	
1	1	0	1	$L.M.\overline{H}$
1	0	1	0	
0	1	1	0	
1	1	1	0	

Canonical form = $\overline{L}.\overline{M}.\overline{H} \vee L.\overline{M}.\overline{H} \vee L.M.\overline{H}$

EXAMPLE 10: A TRUTH TABLE FOR THE BOOLEAN FUNCTION $\overline{H}.(M \Rightarrow L)$ WITH THREE BOOLEAN VARIABLES L, M, AND H. AN OUTPUT COLUMN FOR THE FUNCTION, THE THREE *MINTERMS* FOR THE THREE TRUE OUTPUTS, AND THE CANONICAL FORM OF THIS BOOLEAN FUNCTION of these, and any other Boolean function, is equivalent to a truth table that exhaustively lists all the combinations of true and false, indicating exactly which of these combinations corresponds to (is true of) the entire Boolean function.

Example 10 shows such a truth table, involving three true or false (on or off) variables, a "3-bit" pattern; for three such Boolean variables there are $2^3 = 8$ possible combinations. By convention I use the digits *1* for true and *0* for false. The order of listing these is merely a matter of following a systematic convention. I have listed these 3-bit combinations in increasing numbers of true values: none, one, two, and then three, and within each of these, moving true values from left to right. The Boolean function is defined according to which of the combinations (rows) in the table is deemed true, as indicated in the *output* column (1 for true and 0 for false). Any particular line in a truth table can also be written as a conjunction of literals. For instance the second line would be written $L.\overline{M}.\overline{H}$, which means L is true, and M and H are each false. This conjunction as a whole is true according to this Boolean function, which is why it has a 1 in the output column.

A Boolean function distinguishes exactly which lines in a truth table are true. Yet there are an infinite number of expressions corresponding to these. The Boolean function as a whole can be represented by filtering out the lines of the truth table that it excludes (are false), just using the ones it deems true (in the table, these, called *minterms*, are the ones showing 1 in the output column), and writing these as Boolean expressions using literals and complements (like $L.\overline{M}.\overline{H}$, etc.). From this we can derive the *canonical form* (also called *canonical disjunctive normal form*) of the Boolean function.[4] That is, the *canonical form* of the Boolean function is a disjunction of conjunctions (sum of products) where each product term contains each of the literals (or its complement).[5] The Boolean expression thus stated is unique to this Boolean function. That is, among all the Boolean expressions that are equivalent to this Boolean function, the *canonical form* Boolean expression is uniquely derivable and thus can be relied upon to unambiguously represent this Boolean function. Considering he taught mathematics at Princeton and his musical designs consistently rely on Boolean (on/off) states, how could Babbitt be unfamiliar with this meaning of *canonical form*?

An example will clarify how to derive *canonical form*. Three of the 3-bit combinations (rows) are deemed true, as indicated by the 1s in the rightmost column. The *canonical form* of this Boolean function requires us to represent it as a disjunction of three terms corresponding to the three combinations deemed true: $\overline{L}.\overline{M}.\overline{H}$, $L.\overline{M}.\overline{H}$, and $L.M.\overline{H}$. In other words, the Boolean function is equivalent to stating that $\overline{L}.\overline{M}.\overline{H} \vee L.\overline{M}.\overline{H} \vee L.M.\overline{H}$, which is the *canonical form* of that Boolean function.[6] That canonical form stands as a representational token of all the other Boolean expressions equivalent to this Boolean function. Later I'll return to the relevance (for music and music theory) of being able to determine and use a unique representational token for a class of entities deemed somehow equivalent to each other.

Now let's apply this to the registral states of Babbitt's *Canonical Form*. Consider the first three crucial steps in the registral trajectory, diagrammed in Example 11: section 2 (the first point by which an upward trajectory is articulated); section 3 (the point at which all three

Sections			
0 1	2	3	4
		H	H
	M	M	M
L	L	L	

	Cumulative set of registral states at								
	Section 2			Section 3			Section 4		
Octal number	L M H	Output	Minterm	L M H	Output	Minterm	L M H	Output	Minterm
0	0 0 0	1	$\overline{L}.\overline{M}.\overline{H}$	0 0 0	1	$\overline{L}.\overline{M}.\overline{H}$	0 0 0	1	$\overline{L}.\overline{M}.\overline{H}$
1	1 0 0	1	$L.\overline{M}.\overline{H}$	1 0 0	1	$L.\overline{M}.\overline{H}$	1 0 0	1	$L.\overline{M}.\overline{H}$
2	0 1 0	0		0 1 0	0		0 1 0	0	
3	0 0 1	0		0 0 1	0		0 0 1	0	
4	1 1 0	1	$L.M.\overline{H}$	1 1 0	1	$L.M.\overline{H}$	1 1 0	1	$L.M.\overline{H}$
5	1 0 1	0		1 0 1	0		1 0 1	0	
6	0 1 1	0		0 1 1	0		0 1 1	1	$\overline{L}.M.H$
7	1 1 1	0		1 1 1	1	$L.M.H$	1 1 1	1	$L.M.H$
The canonical form's numeric coding	*014*			*0147*			*01467*		

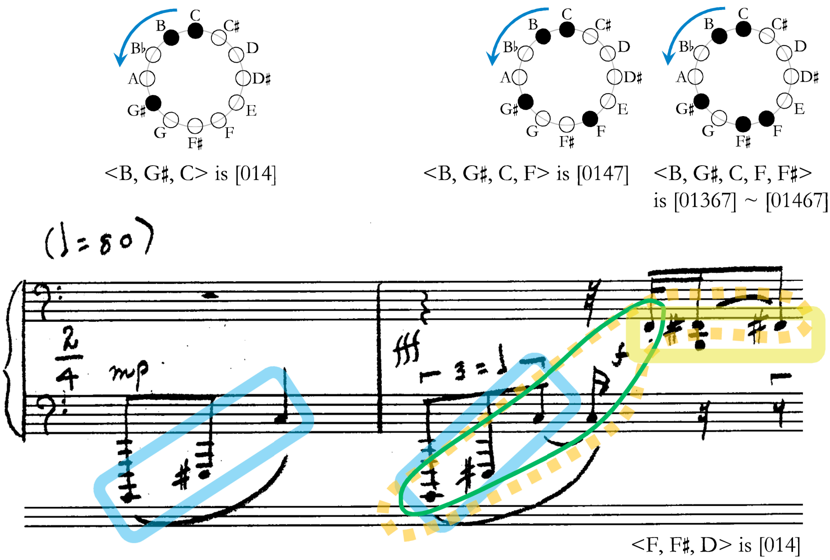
EXAMPLE 11: AT SECTIONS 2, 3, AND 4, THE CUMULATIVE SET OF REGISTRAL STATES UP TO THAT POINT, SHOWN AS *MINTERMS* (FROM WHICH EACH'S CANONICAL FORM IS DERIVED AS A DISJUNCTION) AND THE NUMERIC CODINGS OF THESE (BOTTOM), DERIVE FROM EACH OUTPUT VECTOR'S CORRESPONDING OCTAL NUMBER DIGITS

registers are present); and section 4 (the first registral centroid climax and the only one within the first third of the piece). At each of these points, we could consider the cumulative set of registral states as Boolean functions. The canonical forms of these are:

- $\overline{L}.\overline{M}.\overline{H} \vee L.\overline{M}.\overline{H} \vee L.M.\overline{H}$ (at section 2),
- $\overline{L}.\overline{M}.\overline{H} \vee L.\overline{M}.\overline{H} \vee L.M.\overline{H}. \vee L.M.H$ (at section 3), and
- $\overline{L}.\overline{M}.\overline{H} \vee L.\overline{M}.\overline{H} \vee L.M.\overline{H} \vee L.M.H \vee \overline{L}.M.H$ (at section 4).

These also can be straightforwardly coded by assigning the numbers 0 through 7 (far left column) to the rows of the table, in which case we have 014, 0147, and 01467. These are numeric codings of the canonical forms of the Boolean function's truth table, representing the trajectory of active registers considered cumulatively up to section 4.

Now I would like to turn attention to the pitches in the opening measures of the piece, shown in Example 12. The opening ***mp*** melodic gesture ⟨B_1, G♯$_2$, C3⟩ is set off by being immediately repeated, accelerated to a triplet and now at ***fff***.[7] Its stark ascent forecasts the ascending registral trajectory to come. (I find the spacing of the pitches, clumped toward the bottom, noteworthy.) This rumbling melody continues by descending to F_2, now at a middle dynamic level of ***f***, so that the overall outline of the contour, from low to high to middle (that is, from B_1 up to G♯$_2$ down to F_2) is matched by the trajectory of dynamic levels: from ***mp*** up to ***fff*** down to ***f***. The melody continues with F♯$_2$ (which is accompanied by D_2), at which point the initial trichord pc set type has recurred in a contracted spacing. So let us consider, cumulatively, the set class *prime forms* (trichord type) of the melody at each of these junctures. I mentioned that the opening trichord's pitch spacing is clustered toward the bottom, which is the privileged spacing of our canonical representatives for all our pitch-class set classes, namely *prime form*. Put into prime form, the first melodic trichord pcset ⟨B, G♯, C⟩ is [014]; the first cumulative melodic tetrachord ⟨B, G♯, C, F⟩ is [0147]; and the first cumulative melodic pentachord ⟨B, G♯, C, F, F♯⟩ is [01467]. Now compare these to the numeric codings of the canonical forms of the Boolean function's truth table for the trajectory of active registers considered cumulatively up to section 4. They are the same: [014] followed by [0147] followed by [01467]. In other words, the way that Babbitt's *Canonical Form* begins to traverse the systematically organized space of registral combinations is equivalent to the way it begins to traverse pc space. Of course none of this is recognized without rendering the cumulative registral combinations and cumulative pitch combination in their *canonical* representations.



EXAMPLE 12: THE UNFOLDING MELODIC TRICHORD, TETRACHORD, AND PENTACHORD THAT BEGIN *CANONICAL FORM*, EACH SET CLASS SHOWN IN PRIME FORM FOR T_n-TYPE, MEASURING COUNTERCLOCKWISE

Pragmatism of Systematic Metonymy

There's an interesting lesson in here regarding the general idea of a canonical form, in the sense of a systematically derived token representative for a class of equivalent things. By its very nature, the derivation of a canonical form (whether a pc set class prime form, or a Boolean expression of a set of active registers, or something else) is always influenced by a perspective on the entire space of possibilities, a perspective that might elude us had we not derived the canonical form. The steady build-up of registers, initially from none and ultimately to all ($\overline{L}.\overline{M}.\overline{H}$ to $L.\overline{M}.\overline{H}$ to $L.M.\overline{H}$, and ultimately to $L.M.H$), might seem like a purely incremental increase (and in a sense it is), but in terms of the systematically organized space of possibilities, the moves are initially incremental (clustered at the bottom), but then jumping, leaving gaps in the space ([014] or [0147]), and then slightly filling in ([01467]). Likewise the initial three-note gesture of the piece is a simple rising contour, but when represented in pc prime form (canonical form) it leaves a gap, [014], with the fourth pc creating a further gap, [0147], and the fifth pitch filling that in, [01467], just as the dynamics and pitch contour each create a gap and fill it in (gap-fill: yet another canonical gesture in music, in terms of gestalt theory). Thus to render an entity in canonical form is to gain perspective that is probably absent from our naïve view, the view based on just the chronological flow of entities.

In more general terms, what is a canonical form but a systematic instance of *metonymy*? That is, it's an instance of one member of a class of associated things being understood to represent the entire class of associated things. In mathematics it's a standard, agreed-upon way of writing an equation or other expression (*1* is a canonical form for *2/2*, and for *(4 - 1)/3*, and for (*-5.7/-5.7*), etc.).[8] In mod-12 arithmetic, we use the integer 0 to represent -24, -12, 12, 24, 36 etc. Thus 0 is the canonical form for all those equivalent numbers.

A moment's reflection reveals that systematic metonymy is ubiquitous and indispensable in music. With key signatures we use sharps and flats only once on each staff to canonically represent the equivalent alterations in every octave. For tertian tonal chords, there's a systematic way of deriving the root, which is used to name the chord; a root position chord in some sense canonically represents all its inversional permutations. Movable do solfège takes the pitch labels from C major and uses them to canonically represent equivalent scale degrees in different keys. In Schenkerian analyses, the pitches of the *Urlinie* in the *obligatory register* canonically represent the equivalent pitches in other registers, and diatonic pitches canonically represent their chromatic alterations. Even a I-V-I progression (as occurs in F♯ minor in Babbitt's *Canonical Form*) is taken as a sonic canonical representative of a tonality; it's a canonical way (a canonical form) of expressing a tonality. In musical form analysis the letters ABA stand canonically for all ternary forms. In post-tonal theory, any interval can be reduced to one of six classes and is then canonically represented by one of the digits 1 through 6. Through Morris's (1987; 1993) algorithm, any contour can be reduced to a contour prime, which canonically represents all equivalent contours. And of course any pc set can be reduced algorithmically to a set-class *prime form*, which canonically represents all of its equivalents.

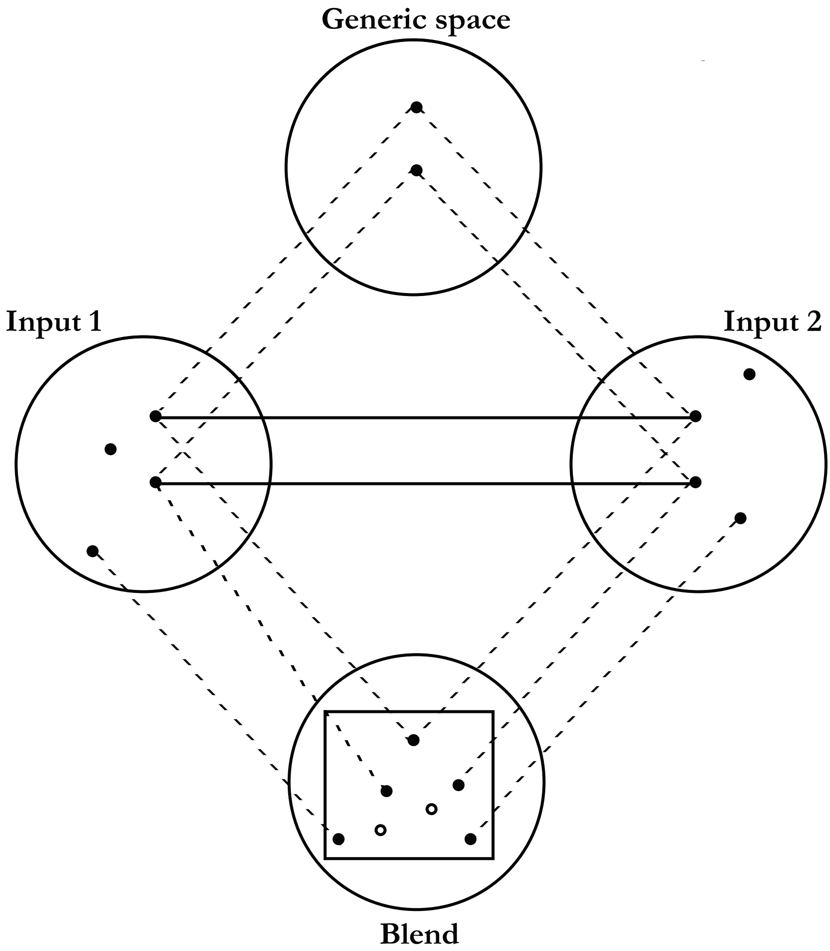
The pragmatic value of canonical forms is legion. Canonicity does not necessarily imply a value judgment. Or putting it differently: the value of canonicity may be in how it facilitates discourse, because it enables certain kinds of metonymy, namely metonymy for certain kinds of very systematic situations, such as we routinely encounter in music. Maggart (2017, 96: paraphrasing and quoting Babbitt [1987]) argues: "Babbitt believed that understanding music leads to an 'understanding of a great many other things.'" The multiple meanings of Babbitt's title *Canonical Form* lead us experientially to this widely relevant bit of pragmatic wisdom.

For this reason, it's sensible to regard Babbitt's double entendre titles not as flippant, prankish, or merely auxiliary, but rather as prompting poetical conceptual blends of which they form a part. Following Maggart's lead, I would like to read more into Babbitt's punning titles, as a window into his compositional poetics, which may offer insights beyond those he verbalized directly in his writings.

Compositional Poetics Expressed Obliquely

Although *Canonical Form* isn't one, several of Babbitt's other titular puns exploit a phonetic ambiguity: *Whirled* (rather than "world") *Series* and *My Complements* (rather than "compliments") *to Roger*. One of the most well-known musical puns is another phonetic pun, the one adopted as the name of the "fabulous four," that is, the Beatles. *The Beetles* was originally suggested as an homage to Buddy Holly's band, the Crickets. John Lennon respelled it as *Beatles* to emphasize the in-vogue style of their music: *beat music*, a late 1950s, early 1960s British post-rock'n'roll genre.[9] It is interesting that in Maggart's classification of Babbitt's titles, *All Set* (1957) is the only pun title before 1969. (We might quibble with Maggart's exclusion of *Post-partitions*, of 1966, which might also be a pun on postpartum; certainly *Semi-Simple Variations* (1956) is a pun on *semi-simple algebraic group*.) Babbitt's punning titles seem to begin in 1969, and are most frequent in the period of 1977 to 1994. Interestingly this period (and 1956–57) is mutually exclusive of the period of the Beatles' existence (1960–70). A coincidence? Perhaps. What I find most intriguing is how, like the Beatles, Babbitt uses puns to subtly draw out and highlight certain potentially neglected facets of his music, or of music in general.

This is particularly noteworthy, as puns are a kind of conceptual blend, which I elsewhere (Mailman 2020) discuss in regard to Babbitt's numerous witty references to facets of tonal music, or in a sense to tonality itself. "Puns are usually described as two meanings being incongruously combined in one and the same utterance," as Lundmark (2003) explains. (Leong [2011] already interprets Babbitt's titular puns as metaphors for musical cross-reference and multiple-function.) Thusly, Babbitt's titling of his compositions parallels the conceptual blends he achieves with pitches—pitches whose configurations have syntactic meaning through their derivation from serial array structure —as well as, incongruously, having "tonal" meaning through their euphony (the blend that I call *portmantonality*).



EXAMPLE 13: TEMPLATE OF FAUCONNIER AND TURNER'S *CONCEPTUAL INTEGRATION NETWORK* (CIN)

A *conceptual blend* is meaning emerging from the integration of two or more *input spaces* achieved by virtue of a subset of *generic* features that the two or more input spaces happen to share. As Fauconnier and Turner (2002) describe it, "a generic mental space maps onto each of the inputs and contains what each of the inputs have in common." Example 13 shows a template for a *conceptual integration network* (CIN), which is what diagrams this situation. Notice the two input spaces enjoy a correspondence by virtue of a subset of their shared features, which constitutes the generic space. These, however, trigger associations with other (not shared) features of each input space, which are thereby organically recruited into the blend. The entire process of integration occurs in three stages: *composition, completion*, and *elaboration*, by which the blend develops emergent structure that is not in the inputs.

An example of such a conceptual blend is diagrammed in Example 14. The blend constitutes, or suggests, a *poetics*, in the sense of a broad conceptual or philosophical outlook beyond the artistic realm, that informs the creation and meaning of the artistic product, but also then reflects back meaning (now exemplified concretely and therefore experientially) outward through the artistic work.

As discussed already, the meaning of the title *Canonical Form* is manifold. First of all, *form* in the title can refer to the form of the musical work. In this case, there are still at least three meanings: First, the form can be archetypical; that is, a form established in the canon of musical works, or in the canon of forms of musical works. (This is Dubiel's "arch form" suggestion.) Second, another meaning is that the form of the musical work is generated by some rule (*canon*) or series of rules (script)—regarding the unfolding of registers, for instance. Third, the musical work is of the genre or procedure ("form") known as a *canon* (implied to be a crab canon), a prototypical kind of imitative polyphony, which almost all twelve-tone music is by default.

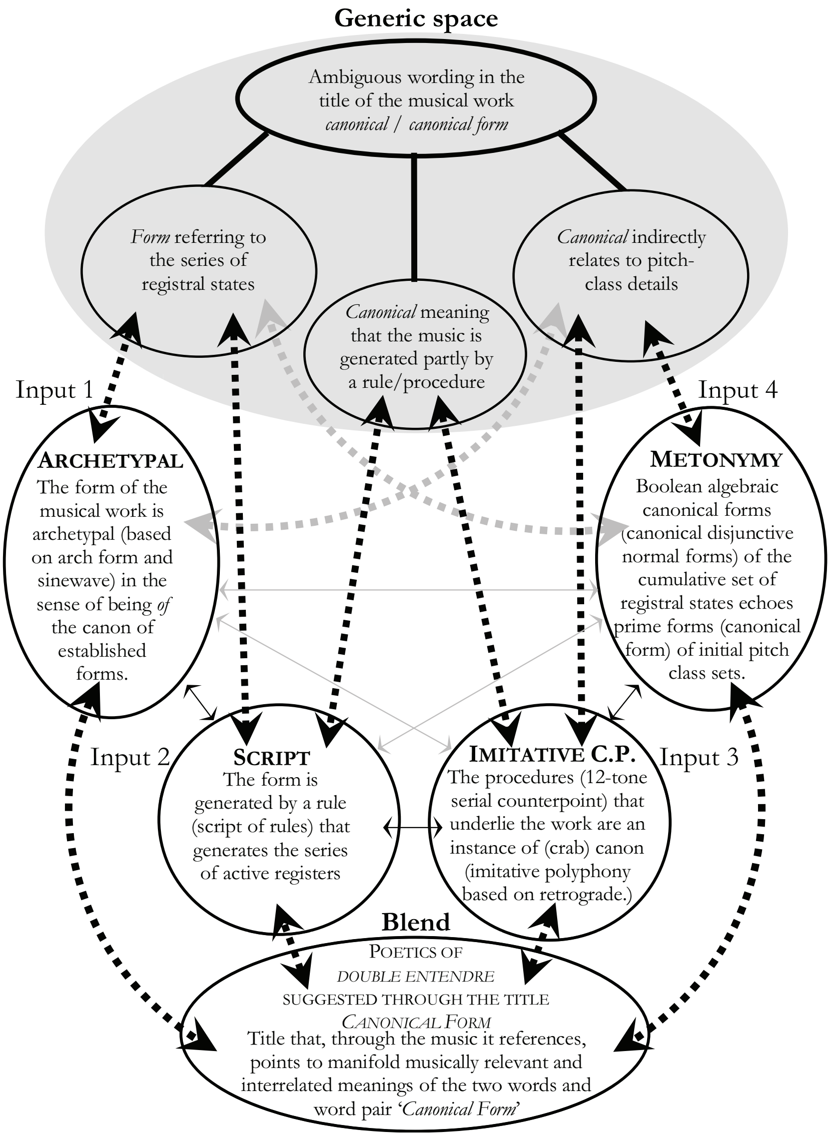
Yet the "form" in the title need not refer to the form of the musical work necessarily; it could refer instead to an instance of systematized metonymy: that is, a systematically derived standard representation, which in various fields of mathematics, for instance Boolean algebra (a branch of mathematical logic), is literally called a "*canonical form*"; thus there is a fourth meaning, which is that the musical work significantly engages a Boolean algebraic *canonical form*. This fourth meaning is especially significant since it suggests a certain pragmatic wisdom about music ontology. As I've explained, all four of these meanings apply to Babbitt's composition by this name. Since the composer of the title deliberately withholds specificity, we are prompted to accept a blend of all of these meanings as constituting the title of his musical work.

Now I want to revisit the canonicity of the accumulative arch form in music that is suggested by the first three and last three sections of Babbitt's piece. Yet this time I consider how it relates more directly to the mathematical notion of canonical form, not in the field of Boolean logic, as already discussed, but rather in the more commonplace area of linear algebra.[10]

Linear algebra concerns methods of solving systems of equations by representing the coefficients as numbers in a matrix each of whose rows corresponds to an equation and whose columns correspond to the variables shared by these equations. An instance of such a system is shown in Example 15 (a) and its matrix representation in Example 15 (b). In linear algebra, canonical form denotes a particularly useful transformation of a matrix of numbers into a diagonal configuration. The most basic type of canonical form in linear algebra is *row canonical form*, often called *row reduced echelon form* (RREF).[11] The word *echelon* means a staircase pattern or gradated steps, as witnessed in the formation of military troops, ships, or planes, the flight configuration of a flock of geese, or the levels in a social or managerial organizational hierarchy. In all these cases the echelon pattern is canonical. An echelon matrix is one that exhibits the staircase pattern as a lower left triangle of zeros, such as in stages (c), (d), and (e) of Example 15. The canonical echelon shape not only visually resembles a significant formal aspect of Babbitt's piano work but also, as I will explain, adds new resonance to the traditional musical form convention that Babbitt's work engages.

Returning to Example 15, focusing on the left column, observe in (a) that there are three equations involving three variables x, y, and z. The matrix at (b) puts the right sides of the equations into its rightmost column and transfers its coefficients from (a) correspondingly. To transform the matrix into *row canonical form* is essentially an algorithmic method of doing the familiar Gaussian elimination, whereby we start by solving for one variable in terms of the others, then use back-substitution to plug this into one of the other equations, and thereby, through a kind of incremental bootstrapping, solve for all the variables one after another.

In the algorithmic guise, we do this by transforming each row of the matrix until its lower left triangle is exclusively filled with zeros, its NW-SE diagonal is exclusively 1s, and finally so that the triangle above the 1s is also exclusively zeros, at which point we can see the solution for each variable in the entries of the rightmost column. For instance, at stage (c) we witness that by using the first row and second row to transform the second row and third row, we arrive at the solution for z.



EXAMPLE 14: *CONCEPTUAL INTEGRATION NETWORK* (CIN) REGARDING THE MULTIPLE MEANINGS OF THE TITLE *CANONICAL FORM*

(a)
$$\begin{aligned} x - 2y + 2z &= 6 \\ 2x - 5y + 7z &= 20 \\ x + y - 5z &= -12 \end{aligned}$$

Write the coefficients of the system of equations as a 3x4 matrix.

(b)
$$\left(\begin{array}{ccc|c} 1 & -2 & 2 & 6 \\ 2 & -5 & 7 & 20 \\ 1 & 1 & -5 & -12 \end{array}\right)$$

Subtract twice the 1st row from the 2nd, and negate the 2nd.
Subtract the 1st row from the 3rd, and thrice the 2nd row from the 3rd.
Divide the 3rd row in half. This provides the solution for z: **z=3**.

(c)
$$\left(\begin{array}{ccc|c} \mathbf{1} & -2 & 2 & 6 \\ 0 & \mathbf{1} & -3 & -8 \\ 0 & 0 & \mathbf{1} & 3 \end{array}\right)$$

Add thrice the 3rd row to the 2nd.
This produces the solution for y: **y = 1**.

(d)
$$\left(\begin{array}{ccc|c} \mathbf{1} & -2 & 2 & 6 \\ 0 & \mathbf{1} & 0 & 1 \\ 0 & 0 & \mathbf{1} & 3 \end{array}\right)$$

Subtract twice the 3rd row from the 1st.
Add twice the 2nd row to the 1st.
This produces the solution for x: **x=2**.

(e)
$$\left(\begin{array}{ccc|c} \mathbf{1} & 0 & 0 & 2 \\ 0 & \mathbf{1} & 0 & 1 \\ 0 & 0 & \mathbf{1} & 3 \end{array}\right)$$

The unit coefficient in each column (with zeros elsewhere) indicates that the far right column lists the solutions to the three variables x, y, and z.

(f)
$$\begin{aligned} x \quad &= 2 \\ y \quad &= 1 \\ z \quad &= 3 \end{aligned}$$

(g)
$$\begin{array}{ccc} x & 2x & x \\ + & - & - \\ y & 5y & 2y \\ - & + & + \\ 5z & 7z & 2z \\ = & = & = \\ -12 & 20 & 6 \end{array}$$

(h)
$$\left(\begin{array}{ccc} 1 & 2 & 1 \\ 1 & 5 & -2 \\ -5 & 7 & 2 \\ \hline -12 & 20 & 6 \end{array}\right)$$

(i)
$$\left(\begin{array}{ccc} 0 & 0 & \mathbf{1} \\ 0 & \mathbf{1} & -2 \\ \mathbf{1} & -3 & 2 \\ \hline 3 & -8 & 6 \end{array}\right)$$

(j)
$$\left(\begin{array}{ccc} 0 & 0 & \mathbf{1} \\ 0 & \mathbf{1} & -2 \\ \mathbf{1} & 0 & 2 \\ \hline 3 & 1 & 6 \end{array}\right)$$

(k)
$$\left(\begin{array}{ccc} 0 & 0 & \mathbf{1} \\ 0 & \mathbf{1} & 0 \\ \mathbf{1} & 0 & 0 \\ \hline 3 & 1 & 2 \end{array}\right)$$

(l)
$$\begin{array}{ccc} & & x \\ & y & \\ z & & \\ = & = & = \\ 3 & 1 & 2 \end{array}$$

EXAMPLE 15: LEFT COLUMN: A SYSTEM OF EQUATIONS FOLLOWED BY STAGES OF THE LINEAR ALGEBRAIC METHOD OF *ROW REDUCED CANONICAL FORM*, OFTEN CALLED *ROW REDUCED ECHELON FORM*; RIGHT COLUMN: EACH STAGE ROTATED CLOCKWISE 90 DEGREES

Adding thrice the third row to the second row yields the solution for *y*, at stage (d). Finally, transforming the first row via the second and third yields the *row canonical form* (*row reduced echelon form* or RREF) of the matrix at stage (e), where the solution to all three variables, including *x*, appears down the rightmost column. This is because the coefficients are now all zero except for one column in each row, where the coefficient 1 isolates exactly one variable per row, so that stage (f) is just another way of writing stage (e).

I hope that the incremental steps of the diagonal we see in steps (c), (d), and (e) conjure some familiarity as a musical design, specifically the incremental increase of registers from low, to medium, to high, in Example 3, and moreover as part of the arch design discussed above and shown in Examples 4, 5, 6, and 7. If the orientation of this diagonal looks wrong, consider that the NW-SE diagonal (from high to low registers) constitutes the profile that ends the arch form (for instance in Babbitt's sections 22–24). Moreover, it is purely by convention that equations are written horizontally; the RREF algorithm is achieved just as easily while rotating all the matrices 90 degrees, as shown on the right side of Example 15, as stages (g) through (l), which are equivalent to stages (a) through (f). Now we can see the SW-NE diagonal in stages (i), (j), and (k), corresponding exactly to the climb of registers (from L to M to H) that so distinctively begins Babbitt's *Canonical Form*.

Yet it is not just the visual shape that is significant here, but rather also the broader idea of incremental accumulation. This idea is meaningful as a canonical rhetorical-formal device in music, as discussed above, from which Wagner draws a parallel to a narrative of primordial ooze incrementally generating the full blossoming of life, in which his epic drama unfolds. The accumulation of incrementally more layers, increasing subdivisions, and higher registers paints this picture in *Das Rheingold*'s Prelude. As I mentioned above, Dubiel (1992) in his analysis of Babbitt's *Canonical Form*, mentions that the lowest register is heard by many as "somehow generative of the entire piece," which I believe resonates with the canonical rhetorical-formal convention exploited and developed by Wagner, but also suggests generativity in another sense. Associating this canonical musical design with the title *Canonical Form* (referencing the linear algebraic method) enables Babbitt's composition to paint a picture of an epistemological progression, a metaphor for knowledge being generated.

Although distinct from the narrative arch conjured by the accumulative arch form in music, it nevertheless relates to it, under a wider umbrella. In humanistic terms, isn't the growth of knowledge a later stage and therefore a consequence of the blossoming of life from its initial state of primordial ooze? And historically isn't this often achieved through an accumulation of incremental steps, each building on the previous: solving one problem which serves as a key to unlock yet another? Such an epistemological progression is represented metonymically in the procedure of *row reduced echelon form* (RREF), or *row canonical form*: solving for one variable, as a key to solving for another, until the entire solution is reached. This is the accumulation of knowledge witnessed through the steps in Example 15. Putting the matrix of coefficients into canonical form serves as an incremental method for solving the system of equations, which may stand metonymically for a physical modeling of the world, and thereby stand metonymically for all knowledge. It illustrates how canonical forms can serve as instruments for developing knowledge. And the canonical rhetorical-formal musical design rendered orthochronically resembles the diagonalized configuration seen in the intermediate stages and completed row canonical form matrix. It is an even more nuanced metaphor than Fux's *Steps to Parnassus*—which it somewhat literally relates to. By revealing yet another mode of resonance between math and music, Babbitt illustrates how the diagonal trajectory (in the matrix and in the ascent of pitch registers) and form canonicity may intertwine. The intertwining ties epistemology into his compositional poetics. It also enlists musical creativity into humans' communal epistemological project, by sonically dramatizing its process.

Babbitt's insights into music are, it seems, quite unusual, and so only obliquely conveyed. Redfern (1984) writes that "puns illuminate the nature of language in general." By analogy, musical puns illuminate the nature of music. From Maggart's (2017) assertion that Babbitt believed understanding music leads to an understand of a great many things, this implies that puns expressed through music illuminate the nature of much else besides music. For instance, a canonical representation—or canonicity generally, in music or music theory, for example—rather than implying a value judgment or indicating a hierarchy of importance, can rather serve pragmatically as an instrument for connecting the otherwise disparate, thus fueling the progress of knowledge.

Notes

1. Lawrence Zbikowski (1999; 2018), Michael Spitzer (2018), and others have applied Fauconnier and Mark Turner's conceptual blending theory in their own music analyses. Another more recent phenomenon is the analysis of metonymy, "metaphoric chains," and hyperbole in music, undertaken by Paula Pérez-Sobrino (2014; 2018).
2. To assist in hearing this tonal reading, three audio synchronized scrolling videos are provided. I suggest watching Example 2 video (a-b.1), where the audio is from the score, while watching the voice-leading graph on the bottom, and then watching Example 2 video (a-b.2), where the audio is from the voice-leading graph, while watching the score on the top. I suspect that this cross-observational activity promotes a better intuition of how the score and voice-leading graph relate to one's tonal hearing of the passage. Then watch video a-b-c, in which a deeper-level graph, the shallower-level graph, and the score are played in series. Then listen to Example 2 audio a.1, a.2, and b.
3. For instance, Victor Zuckerkandl (1956, 151–200) interprets musical meter dynamically as waves, somewhat analogous to the periodic vibrations that underlie the experience of pitch.
4. The notion of *canonical form* occurs also in other branches of mathematics besides Boolean logic.
5. See Brian Lawless's (1996) *Fundamental Digital Electronics*, unit 11: "Canonical Forms." Also Bender and Williamson (2005) and McCluskey (1965).
6. Verbally this could be expressed: "Either neither M, L, nor H is true, or L is true, but neither M nor H is, or L and M are true but H is not."
7. Dubiel (1992) also notes that it's cited elsewhere in the piece, most conspicuously in the final section.
8. Other branches of mathematics have their own kinds of *canonical forms* (or *normal forms*); for instance, in predicate calculus *prenex, normal form* is conversion of a formula so that all of its quantifier ($\forall$ and $\exists$) instances appear first, before its quantifier-free part: for instance, $\forall x\, \exists y\, \forall z\, (\varphi(y) \vee (\Psi(z) \Rightarrow \rho(x)))$.

9. Refer to Gilliland (1969), show 27, track 4.

10. One could consult Strang (2016) as an up-to-date approach to this increasingly relevant mathematical subject.

11. A quick introduction to the mechanics of *row canonical form* or *row reduced echelon* form (RREF) is found in chapter 2 of Lipschutz (1997).

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